

Optimizing Relay Precoding for Wireless Coordinated Relaying

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I. INTRODUCTION

Recently there have been extensive studies on cooperative, relay-based transmissions for extending cellular coverage or increasing diversity. Several basic relaying techniques have been introduced, such as amplify-and-forward (AF) [1], [2], decode-and-forward [3], [4] and compress-and-forward [5].

These transmission techniques have been applied in one-way, two-way or multi-way relaying scenarios. There has been a particularly high interest in two-way relaying scenarios [6], [7], [8], [9], [10], where throughput gains have been demonstrated by utilizing the ideas of wireless network coding [11], [12]. The two underlying principles used in designing throughput-efficient schemes with wireless network coding:

- 1) Aggregation of multiple communication flows: instead of transmitting each flow independently, network coding is used where flows are sent/processed jointly;
- 2) Network coding intentionally allows interference and simultaneous usage of the shared wireless medium, leaving to the receivers to remove the adverse impact of interference by using any side information.

Leveraging on these principles, there are proposed schemes with AF relaying in [13], [14] that feature more general traffic patterns compared to the two-way relaying. These schemes are termed *coordinated direct/relay* (CDR) transmissions. The CDR transmission considers scenarios where one direct user (UE) and one relayed UE are served in uplink/downlink. The relayed UE is assumed to have no direct link to the base station (BS) due to large path loss and relies only on the amplified/forwarded signal from the relay in order to decode the signal from the BS. Schemes that are related to some of the schemes have appeared before in [15], [16], [17].

Each user might have a downlink or uplink traffic. Hence, there are different traffic configurations. We focus on one representative traffic type with one relayed uplink UE and one direct downlink UE.

This case displays the merits of analog network coding in a setting that is more general than the usual two-way relay scenario. Furthermore it showcases the principle of overheard information where a node overhears a signal that is not intended to itself and uses it as *a priori* information to cancel interference in an ulterior transmission phase.

In the scheme on Fig. 1, we assume that a relayed UE has one signal to deliver to the BS through the assistance of the relay station, while a direct user wants to receive a signal from the BS. Notice in a conventional wireless cellular system, these signals are sent over two orthogonal uplink and downlink phases for the two separate information flows, respectively. Instead in the CDR system, the BS first sends the signal to the direct UE and simultaneously the relayed UE transmits the signal to the relay station in phase 1. The relay receives two signals: the desired signal from the UE and an interfering signal from the BS. It does not decode the signals but instead forward them in phase 2 using the principle of analog network coding. The simultaneous two-flow transmissions improve the *spectral efficiency* compared to the conventional method. The key points are the BS can use the *a priori* information to perform self-interference cancellation and enable interference-free reception and decoding; the direct UE can use the overheard information in phase 2 to help decoding the desired signal.

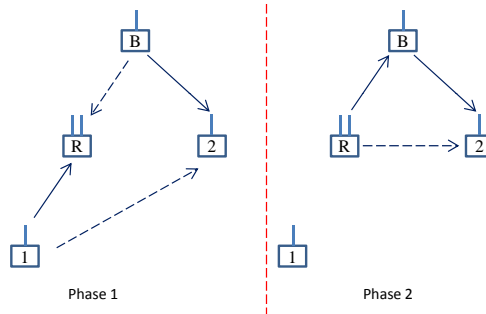


Figure 1. CDR MIMO System Model.

In the works that deal with the CDR transmission, the relay has a central role in managing the interference. Therefore, in this work we investigate the qualitative changes and the performance improvements that arise when the relay node in the CDR schemes is equipped with multiple antennas. Differently from the previous works, the usage of multiple antennas at the relay permits to manage the interference and boost the overall system performance through beamforming. This is a significant conceptual difference compared to the original CDR schemes, while the usage of multiple antennas at the BS and the UEs is a clear future work. We consider AF operation at the relay, assuming that the relay and the reception nodes have a perfect channel state information (CSI). Our objective is to maximize the achievable sum-rate of the system. Our design shows that the overall system performance is improved by allowing the relay

beamformer to deliver the interfered signal to both the BS and the direct UE in phase 2. Meanwhile, the BS completely cancels the self-interference; the direct UE applies linear interference minimization receiver to decode the desired signal.

We propose three low-complexity algorithms to approach an upper bound on the sum-rate, namely the adaptive subspace averaging algorithm, the power iteration algorithm and the linear space spanning algorithm. Their performance is shown via simulations to be close to the tight upper bound on the sum-rate. The gain via possessing multiple relay antennas is also shown compared to the original CDR transmission.

Notation: We use uppercase and lowercase boldface letters to represent matrices and vectors, respectively. \otimes refers to the Kronecker product and $\|\cdot\|_F^2$ denotes the Frobenius norm of a matrix. \mathbb{I} is the identity matrix.

II. SYSTEM OVERVIEW

The basic setup is the scenario in Fig. 1 with one BS, one relay, and two UEs. The relay is equipped with M antennas. The BS and the UEs are equipped with one antenna only. The transmission from the relayed UE to the relay has the same duration as the transmission from the relay to the BS. The relay is deployed to help the relayed UE which has no direct link to the BS due to large path loss.

We consider the multi-antenna relay beamforming design where there are two information flows: the relayed UE (UE 1) delivers x_1 to the BS and the BS transmits x_2 to the direct UE (UE 2). The conventional system will create two orthogonal transmissions for separate information flows, while the CDR system enables simultaneous transmissions and thus improves the system spectral efficiency. Here we illustrate the two-phase CDR transmission in Fig. 1. In the first slot, UE 1 transmits x_1 to the relay and the BS delivers x_2 to UE 2 simultaneously. At the same time, UE 2 overhears the signal from UE 1 and the relay also receives the signal from the BS. Then the relay forwards the received physical layer network-coded signal to both the BS and the direct UE in the second slot. The BS has the capability to use the *a priori* information it transmits in the first slot to perform self-interference cancellation. In summary, this gives a more general traffic pattern compared to the two-way relaying.

In this CDR system, each channel is assumed to be an independent complex Gaussian random variable with zero mean and unit variance. All links are assumed to be static within the two slots. Assume P to be the transmit power of the BS and each UE, the received signals at the relay and UE 2 in the first slot are

$$\begin{aligned}
\mathbf{y}_R &= \sqrt{P}\mathbf{h}_{R1}x_1 + \sqrt{P}\mathbf{h}_{RB}x_2 + \mathbf{n}_R \\
y_2[1] &= \sqrt{P}h_{21}x_1 + \sqrt{P}h_{2B}x_2 + n_2[1]
\end{aligned} \tag{1}$$

where \mathbf{n}_R is the complex white Gaussian noise vector at the relay with the covariance matrix $\mathbb{E}[\mathbf{n}_R\mathbf{n}_R^H] = \mathbb{I}$ and $n_2[1]$ is the complex white Gaussian noise variable at UE 2 in the first slot with unit variance¹. The received signals at the BS and UE 2 in the second slot are

$$\begin{aligned}
y_B &= \mathbf{h}_{BR}\mathbf{x}_R + n_B \\
y_2[2] &= \mathbf{h}_{2R}\mathbf{x}_R + n_2[2]
\end{aligned} \tag{2}$$

where the signal vectors transmitted from the relay is in the form $\mathbf{x}_R = \mathbf{W}\mathbf{y}_R$ with \mathbf{W} being the $M \times M$ relay beamforming matrix. At the relay, \mathbf{W} is used here to linearly process $M \times 1$ received signal vector and form the $M \times 1$ transmit signal vector without loss of generality. n_B and $n_2[2]$ are the complex white Gaussian noise variables with unit variance each at the BS and UE 2 respectively. The total relay power is constrained not to exceed a power budget $\mathbb{E}[\mathbf{x}_R^H\mathbf{x}_R] = P(\mathbf{h}_{RB}^H\mathbf{W}^H\mathbf{W}\mathbf{h}_{RB} + \mathbf{h}_{R1}^H\mathbf{W}^H\mathbf{W}\mathbf{h}_{R1}) + \|\mathbf{W}\|_F^2 \leq P_R$.

III. ACHIEVABLE SUM-RATE MAXIMIZATION FOR COORDINATED RELAY BEAMFORMING

From the previous illustration, the relay has the capability to beamform the received network-coded signal and forwards the beamformed signal to both the BS and the direct UE in the second phase of the CDR transmission. Via sum-rate maximal relay beamforming design, the overall CDR system performance is enhanced by allowing the relay to balance between maximizing the rate of the transmission from the relayed UE to the BS and rate of the transmission from the BS to the direct UE. The central role of the relay in balancing the two information flows can be observed in Fig. ?? . In this section, we focus on the problem of achievable sum-rate maximization subjecting to the total relay power constraint. Using the relay beamforming matrix as the design parameter, the problem is shown to be equivalent to maximizing the product of two fractional quadratic functions. A tight upper performance bound on the sum-rate will be given first and three low-complexity solutions will be provided to approach the optimal solution of the non-convex problem.

¹We assume the variance of each noise component is normalized.

A. Problem Formulation

The sum-rate maximization problem can be formulated as

$$\begin{aligned} & \arg \max_{\mathbf{W}} (R_1 + R_2) \\ & s.t. \ P(\mathbf{h}_{RB}^H \mathbf{W}^H \mathbf{W} \mathbf{h}_{RB} + \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{W} \mathbf{h}_{R1}) \\ & \quad + \|\mathbf{W}\|_F^2 \leq P_R \end{aligned}$$

where R_1 and R_2 denote the rate expressions for the transmission of x_1 and x_2 , respectively. The rate expression for each information flow can be written as $R_1 = \frac{1}{2} \log_2(1 + \text{SNR}_1)$ and $R_2 = \frac{1}{2} \log_2(1 + \text{SINR}_2)$ where SNR_1 is the SNR expression for the BS to decode x_1 and SINR_2 is the SINR expression for the direct UE to decode x_2 . And the factor $\frac{1}{2}$ is due to the two time slots transmission duration. This is because from the analog network coding principle, x_2 is known *a priori* at the BS and the related interference is mitigated via the self-interference cancellation process. Therefore, there is no interference when the BS wants to decode x_1 . Notice we then use linear receivers in the CDR system to decode the desirable signals at the BS and the direct UE. Using the monotonicity of the log function, the sum-rate maximization problem can be rewritten as

$$\begin{aligned} & \arg \max_{\mathbf{W}} [(1 + \text{SNR}_1)(1 + \text{SINR}_2)] \\ & s.t. \ P(\mathbf{h}_{RB}^H \mathbf{W}^H \mathbf{W} \mathbf{h}_{RB} + \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{W} \mathbf{h}_{R1}) \\ & \quad + \|\mathbf{W}\|_F^2 \leq P_R. \end{aligned} \tag{3}$$

We first take a look at the SNR and SINR expressions for both UEs. For the BS, after self-interference cancellation, we will have $\hat{y}_B = \sqrt{P} \mathbf{h}_{BR} \mathbf{W} \mathbf{h}_{R1} x_1 + \mathbf{h}_{BR} \mathbf{W} \mathbf{n}_R + n_B$. Then the SNR at the BS is expressed as

$$\text{SNR}_1 = \frac{P \mathbf{h}_{BR} \mathbf{W} \mathbf{h}_{R1} \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{h}_{BR}^H}{\mathbf{h}_{BR} \mathbf{W} \mathbf{W}^H \mathbf{h}_{BR}^H + 1}.$$

Meanwhile, the direct UE uses $y_2[1]$ from the first slot and $y_2[2]$ from the second slot to form a virtual 2-antenna received signal vector $\mathbf{y}_2 = \begin{bmatrix} y_2[1] & y_2[2] \end{bmatrix}^T$

$$\mathbf{y}_2 = \begin{bmatrix} \sqrt{P}h_{2B} \\ \sqrt{P}\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{RB} \end{bmatrix} x_2 + \begin{bmatrix} \sqrt{P}h_{21} \\ \sqrt{P}\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{R1} \end{bmatrix} x_1 + \begin{bmatrix} n_2[1] \\ \mathbf{h}_{2R}\mathbf{W}\mathbf{n}_R + n_2[2] \end{bmatrix}.$$

Then the direct UE wants to estimate the desired signal x_2 and x_1 is the interference from the other information flow. We use simple zero forcing (ZF) receiver at the direct UE to aim for a low computational complexity [18], [19]. The corresponding SINR at UE 2 is derived as

$$\text{SINR}_2 = \frac{P \|\mathbf{h}_{2B}\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{R1} - h_{21}\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{RB}\|_2^2}{|h_{21}|^2 (\mathbf{h}_{2R}\mathbf{W}\mathbf{W}^H\mathbf{h}_{2R}^H + 1) + \|\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{R1}\|_2^2}.$$

The following lemma summarizes the main result of the problem formulation and is proved in the Appendix.

Lemma 1. *The sum-rate maximization beamforming design is equivalent to maximizing the product of two fractional quadratic functions*

$$\arg \max_{\tilde{\mathbf{w}}} G(\tilde{\mathbf{w}}) = \arg \max_{\tilde{\mathbf{w}}} \left[\frac{\tilde{\mathbf{w}}^H \mathbf{A} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{B} \tilde{\mathbf{w}}} \times \frac{\tilde{\mathbf{w}}^H \mathbf{C} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{D} \tilde{\mathbf{w}}} \right] \quad (4)$$

where matrices \mathbf{A} , \mathbf{B} , \mathbf{C} , and \mathbf{D} are not dependent on $\tilde{\mathbf{w}}$. Then $\tilde{\mathbf{w}}$ is scaled to fulfill the power constraint $\tilde{\mathbf{w}}^H \tilde{\mathbf{w}} = P_R$.

In the following, a tight upper bound on the sum-rate is derived first and three achievable sum-rate maximization relay beamforming algorithms will be proposed.

B. Upper Bound

A tight upper bound on the sum-rate for this CDR system is derived in this section. An upper bound on the sum-rate for the two-way multi-antenna AF relay system with single-antenna UEs is given in [20]. Following [20], we consider the artificial case where the relay could use a beamforming matrix \mathbf{W}_1 optimized for transmission to the relayed UE and a different beamforming matrix \mathbf{W}_2 optimized for transmission to the direct UE. In reality, we have a broadcast transmission and the same beamforming matrix is used for both transmissions. From the Appendix, we know that it is optimal for the relay to transmit at full power. An upper bound on the sum-rate is

$$\begin{aligned}
& \max_{\mathbf{W}_1, \mathbf{W}_2} \frac{1}{2} \log_2 [1 + \text{SNR}_1(\mathbf{W}_1)] + \frac{1}{2} \log_2 [1 + \text{SINR}_2(\mathbf{W}_2)] \\
& s.t. \quad P(\mathbf{h}_{R1}^H \mathbf{W}_1^H \mathbf{W}_1 \mathbf{h}_{R1} + \mathbf{h}_{RB}^H \mathbf{W}_2^H \mathbf{W}_2 \mathbf{h}_{RB}) \\
& \quad + \kappa_1 \|\mathbf{W}_1\|_F^2 + \kappa_2 \|\mathbf{W}_2\|_F^2 = P_R
\end{aligned} \tag{5}$$

where $\text{SNR}_1(\mathbf{W}_1)$ is a function of \mathbf{W}_1 and $\text{SINR}_2(\mathbf{W}_2)$ is a function of \mathbf{W}_2 . κ_1 and κ_2 are non-negative and fulfilling $\kappa_1 + \kappa_2 = 1^2$. For the two different beamformers \mathbf{W}_1 and \mathbf{W}_2 , $\kappa_1 \|\mathbf{W}_1\|_F^2$ and $\kappa_2 \|\mathbf{W}_2\|_F^2$ represent the corresponding two fractions of power related to the noise enhancement in the AF relaying, respectively. Denote $R(\kappa_1, \kappa_2)$ to be the solution to (5). This upper bound can be tightened by minimizing $R(\kappa_1, \kappa_2)$ over all feasible values of κ_1 and κ_2 . When κ_1 and κ_2 are given, (5) can be equivalently decomposed into two independent sub problems in (6) where P_1 and P_2 are the total relay power consumptions of the beamformers \mathbf{W}_1 and \mathbf{W}_2 , respectively. Therefore, $P_1 + P_2 = P_R$. $R(\kappa_1, \kappa_2)$ is then derived via $R_1(\kappa_1, P_1) + R_2(\kappa_2, P_2)$ maximization over all the feasible pairs of P_1 and P_2 .

$$\begin{aligned}
R_1(\kappa_1, P_1) &= \max_{\mathbf{W}_1} \frac{1}{2} \log_2 [1 + \text{SNR}_1(\mathbf{W}_1)] \\
& s.t. \quad P \mathbf{h}_{R1}^H \mathbf{W}_1^H \mathbf{W}_1 \mathbf{h}_{R1} + \kappa_1 \|\mathbf{W}_1\|_F^2 \leq P_1 \\
R_2(\kappa_2, P_2) &= \max_{\mathbf{W}_2} \frac{1}{2} \log_2 [1 + \text{SINR}_2(\mathbf{W}_2)] \\
& s.t. \quad P \mathbf{h}_{RB}^H \mathbf{W}_2^H \mathbf{W}_2 \mathbf{h}_{RB} + \kappa_2 \|\mathbf{W}_2\|_F^2 \leq P_2.
\end{aligned} \tag{6}$$

The tightest upper bound is R_{UB}

$$R_{\text{UB}} = \min_{\kappa_1 + \kappa_2 = 1} \max_{P_1 + P_2 = P_R} R_1(\kappa_1, P_1) + R_2(\kappa_2, P_2).$$

According to the derivations in the Appendix, the solutions to the two sub-problems can be derived via the generalized Rayleigh quotient. However, no closed form solution exists for R_{UB} and numerical search over κ_1 , κ_2 , P_1 and P_2 is required. This upper bound on the sum-rate will be used to characterize the loss resulting from the use of suboptimal optimization methods discussed in the following.

²If we additionally impose the constraint that $\mathbf{W}_1 = \mathbf{W}_2 = \mathbf{W}$, the solution \mathbf{W} will give the exact maximal sum-rate of the CDR system.

C. Beamforming Optimization Methods

Since the achievable sum-rate maximization problem (4) is a non-convex problem [21], where global optimum solution is difficult to obtain within reasonable computation time. This optimization problem has generally no closed form solution. Well-known iterative methods can be applied such as simulated annealing, genetic and branch-and-bound³ algorithms which require very high computational load. We focus on low-complexity algorithms to avoid prohibitively high computational complexity. The proposals will be demonstrated via simulations in Section IV to be near-optimal solutions.

1) *Adaptive Subspace Averaging Algorithm (ASS)*: We first propose a suboptimal solution based on the subspace averaging approach [25], [26]. The concept of subspace averaging was introduced first in a covariance matrix suboptimal estimation with a fixed number of dominating eigenvalues.

We use a simple but loose upper bound to form our design. The cost function in (4) can be first approximated via using the inequality of arithmetic and geometric means

$$\frac{\tilde{\mathbf{w}}^H \mathbf{A} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{B} \tilde{\mathbf{w}}} \times \frac{\tilde{\mathbf{w}}^H \mathbf{C} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{D} \tilde{\mathbf{w}}} \leq \frac{\left(\frac{\tilde{\mathbf{w}}^H \mathbf{A} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{B} \tilde{\mathbf{w}}} + \frac{\tilde{\mathbf{w}}^H \mathbf{C} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{D} \tilde{\mathbf{w}}} \right)^2}{2}.$$

An adaptive real value α ($0 \leq \alpha \leq 1$) is then introduced to form the averaging adaptation with respect to α , which will not change the optimization of the cost function in (4). We will obtain

$$\alpha \frac{\tilde{\mathbf{w}}^H \mathbf{A} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{B} \tilde{\mathbf{w}}} \times (1 - \alpha) \frac{\tilde{\mathbf{w}}^H \mathbf{C} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{D} \tilde{\mathbf{w}}} \leq \frac{\left[\alpha \frac{\tilde{\mathbf{w}}^H \mathbf{A} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{B} \tilde{\mathbf{w}}} + (1 - \alpha) \frac{\tilde{\mathbf{w}}^H \mathbf{C} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{D} \tilde{\mathbf{w}}} \right]^2}{2}$$

We denote $g_1(\tilde{\mathbf{w}}) = \frac{\tilde{\mathbf{w}}^H \mathbf{A} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{B} \tilde{\mathbf{w}}}$ and $g_2(\tilde{\mathbf{w}}) = \frac{\tilde{\mathbf{w}}^H \mathbf{C} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{D} \tilde{\mathbf{w}}}$. We denote $\arg \max_{\tilde{\mathbf{w}}} [\alpha g_1(\tilde{\mathbf{w}}) + (1 - \alpha) g_2(\tilde{\mathbf{w}})]$ to be the approximated objective function for the adaptive subspace averaging (ASA) algorithm. The two terms in the approximated objective can be rewritten as

$$\begin{aligned} g_1(\tilde{\mathbf{w}}) &= \frac{\tilde{\mathbf{u}}^H \mathbf{B}^{-\frac{1}{2}} \mathbf{A} \mathbf{B}^{-\frac{1}{2}} \tilde{\mathbf{u}}}{\tilde{\mathbf{u}}^H \tilde{\mathbf{u}}}, & \tilde{\mathbf{u}} &= \mathbf{B}^{\frac{1}{2}} \tilde{\mathbf{w}} \\ g_2(\tilde{\mathbf{w}}) &= \frac{\tilde{\mathbf{v}}^H \mathbf{D}^{-\frac{1}{2}} \mathbf{C} \mathbf{D}^{-\frac{1}{2}} \tilde{\mathbf{v}}}{\tilde{\mathbf{v}}^H \tilde{\mathbf{v}}}, & \tilde{\mathbf{v}} &= \mathbf{D}^{\frac{1}{2}} \tilde{\mathbf{w}}. \end{aligned} \quad (7)$$

We notice that (7) projecting the dominating components of $g_1(\tilde{\mathbf{w}})$ and $g_2(\tilde{\mathbf{w}})$ onto the subspaces spanned by $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{v}}$, respectively. The two individual equivalent eigenvalue decompositions to (7) are

$$g_1(\tilde{\mathbf{w}}) = \frac{\tilde{\mathbf{w}}^H \mathbf{B}^{-1} \mathbf{A} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \tilde{\mathbf{w}}}, \quad g_2(\tilde{\mathbf{w}}) = \frac{\tilde{\mathbf{w}}^H \mathbf{D}^{-1} \mathbf{C} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \tilde{\mathbf{w}}}.$$

³The branch-and-bound method [22], [23], [24] can be applied to solve problem (4) and obtain the global optimal solution. It will be treated in a future work to further evaluate the proposed algorithms.

Therefore, the adaptive subspace averaging of the approximated objective function is expressed as $\mathbf{\Pi} = \alpha \mathbf{B}^{-1} \mathbf{A} + (1 - \alpha) \mathbf{D}^{-1} \mathbf{C}$ where $\alpha (0 \leq \alpha \leq 1)$ is the adaptive parameter. The introduction of the adaptive parameter is the novelty of this ASA algorithm since the previous application in [26] uses a fixed $\alpha = 0.5$. From the subspace averaging of the approximated objective, the suboptimal ASA solution $\tilde{\mathbf{w}}$ can be derived by solving the following problem:

$$\max_{\alpha} \left\{ \max_{\tilde{\mathbf{w}}} [\tilde{\mathbf{w}}^H \mathbf{\Pi} \tilde{\mathbf{w}}] \right\} \text{ s.t. } \tilde{\mathbf{w}}^H \tilde{\mathbf{w}} = P_R, 0 \leq \alpha \leq 1$$

where the solution is obtained via the principal eigenvector of $\alpha_{opt} \mathbf{B}^{-1} \mathbf{A} + (1 - \alpha_{opt}) \mathbf{D}^{-1}$ and then scaled to fulfill the power constraint $\tilde{\mathbf{w}}^H \tilde{\mathbf{w}} = P_R$. The optimal α_{opt} is obtained via a grid search followed by the Nelder-Mead method⁴ [27]. Although this algorithm relies on an approximated objective function of the cost function in (4), it will be shown by simulations to provide sum-rate results approaching the upper bound.

2) *Power Iteration Algorithm (PIA)*: The second algorithm attempts to obtain a solution to the Karush-Kuhn-Tucker (KKT) conditions. The first order necessary condition $\frac{\partial G(\tilde{\mathbf{w}})}{\partial \tilde{\mathbf{w}}} = 0$ leads to

$$G(\tilde{\mathbf{w}}) [(\tilde{\mathbf{w}}^H \mathbf{B} \tilde{\mathbf{w}}) \mathbf{D} + (\tilde{\mathbf{w}}^H \mathbf{D} \tilde{\mathbf{w}}) \mathbf{B}] \tilde{\mathbf{w}} = [(\tilde{\mathbf{w}}^H \mathbf{C} \tilde{\mathbf{w}}) \mathbf{A} + (\tilde{\mathbf{w}}^H \mathbf{A} \tilde{\mathbf{w}}) \mathbf{C}] \tilde{\mathbf{w}}$$

which can be rewritten as $G(\tilde{\mathbf{w}}) \mathbf{V}(\tilde{\mathbf{w}}) \tilde{\mathbf{w}} = \mathbf{R}(\tilde{\mathbf{w}}) \tilde{\mathbf{w}}$. Notice $\mathbf{V}(\tilde{\mathbf{w}})$ and $\mathbf{R}(\tilde{\mathbf{w}})$ depend on the unknown $\tilde{\mathbf{w}}$. If the dependence could be removed, then the optimizer $\tilde{\mathbf{w}}$ is obviously the eigenvector corresponding to the largest eigenvalue of the matrix $\mathbf{V}^{-1} \mathbf{R}$. However, eigenvalue decomposition of the matrix $[\mathbf{V}(\tilde{\mathbf{w}})]^{-1} \mathbf{R}(\tilde{\mathbf{w}})$ can not be accomplished in closed form. Consequently, we propose a power iteration algorithm (PIA) which finds the principal eigenvector corresponding to the maximum eigenvalue in $[\mathbf{V}(\tilde{\mathbf{w}})]^{-1} \mathbf{R}(\tilde{\mathbf{w}})$ iteratively. This algorithm comes from the power iteration idea in [28], [29]. This proposed iterative algorithm is described in Algorithm 1. Then the beamforming solution is scaled to meet the power constraint $\tilde{\mathbf{w}}^H \tilde{\mathbf{w}} = P_R$.

Since the optimization problem is non-convex, the proposed algorithm cannot guarantee convergence. Extensive simulations have demonstrated the convergence property: 20 iterations appear to be sufficient. In addition, PIA provides a sub-optimal solution giving near-optimal sum-rate results which as shown in Section IV.

3) *Linear Space Spanning Algorithm (LSS)*: We know from Lemma 1 that solving the sum-rate maximizing problem is equivalent to maximizing $g(\tilde{\mathbf{w}}) = g_1(\tilde{\mathbf{w}})g_2(\tilde{\mathbf{w}})$ jointly. The two beamforming vectors $\tilde{\mathbf{w}}_1$ and $\tilde{\mathbf{w}}_2$ maximizing $g_1(\tilde{\mathbf{w}}_1)$ and $g_2(\tilde{\mathbf{w}}_2)$ separately could be straightforwardly obtained.

⁴In Matlab, the Nelder-Mead method is implemented via the “fminsearch” function.

Algorithm 1 Power Iteration Algorithm (PIA)

Initialization: set $n = 0$ and $\tilde{\mathbf{w}}^{(0)} = \tilde{\mathbf{w}}^{(\text{init})}$

iterate

 update $n = n + 1$

 1) $\mathbf{q}^{(n)} = [\mathbf{V}(\tilde{\mathbf{w}}^{(n)})]^{-1} \times [\mathbf{R}(\tilde{\mathbf{w}}^{(n)})] \tilde{\mathbf{w}}^{(n)}$

 2) $\tilde{\mathbf{w}}^{(n+1)} = \sqrt{P_R} \mathbf{q}^{(n)} / \|\mathbf{q}^{(n)}\|_2$

until $G(\tilde{\mathbf{w}}^{(n+1)})$ or sum-rate convergence

The third low-complexity suboptimal solution is proposed based on this observation to optimally combine the two vectors. The solution is chosen to lie in the linear space spanned by $\tilde{\mathbf{w}}_1$ and $\tilde{\mathbf{w}}_2$, $\tilde{\mathbf{w}}_{LSS} = a\tilde{\mathbf{w}}_1 + b\tilde{\mathbf{w}}_2$ where a and b are real value parameters. This algorithm is termed to be the linear space spanning (LSS) algorithm. It is obvious to see that any scaling of a does not change the $[g_1(\tilde{\mathbf{w}}_{LSS}) \ g_2(\tilde{\mathbf{w}}_{LSS})]$ maximization. Therefore, $\tilde{\mathbf{w}}_{LSS}$ is further simplified by letting $a = 1$ and $\tilde{\mathbf{w}}_{LSS} = \tilde{\mathbf{w}}_1 + b\tilde{\mathbf{w}}_2$. It is worth pointing out that the sum-rate maximization problem is transformed into a maximization of a scalar-valued nonlinear function $g(b)$ over one real parameter without constraints. Simulations show there is a global maximal for $g(b)$. Again, a grid search using the Nelder-Mead method is applied to efficiently solve the problem. The obtained beamforming solution should be scaled to satisfy the relay power constraint in the end.

4) *Computational Complexity:* In the adaptive subspace averaging and linear space spanning algorithms, the optimization is over one real parameter only, the computational complexity is lower than e.g. the branch-and-bound algorithm. In the power iteration algorithm, the fast convergence behavior guarantees relatively low computational complexity.

IV. NUMERICAL RESULTS

In this section, we present simulation results for the sum-rate. We assume the relay and the BS have the same transmit power, i.e. $P_R = P$. The relay beamforming designs targeting either SNR_1 maximization or SINR_2 maximization are also included. In addition, to assess the effect of linear relay beamforming, the trivial pure amplification relaying $\mathbf{W} = \sqrt{P_R / (P\mathbf{h}_{RB}^H \mathbf{h}_{RB} + P\mathbf{h}_{R1}^H \mathbf{h}_{R1} + \|\mathbb{I}\|_F^2)} \mathbb{I}$ is also considered. The benchmark with single antenna relay is included to evaluate the gain from using multiple relay antennas.

$$\arg \max_{\mathbf{W}} \left(1 + \frac{P \|\mathbf{h}_{BR} \mathbf{W} \mathbf{h}_{R1}\|_2^2}{\mathbf{h}_{BR} \mathbf{W} \mathbf{W}^H \mathbf{h}_{BR}^H + 1} \right) \times \left(1 + \frac{P \|h_{2B} \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{R1} - h_{21} \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{RB}\|_2^2}{|h_{21}|^2 (\mathbf{h}_{2R} \mathbf{W} \mathbf{W}^H \mathbf{h}_{2R}^H + 1) + \|\mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{R1}\|_2^2} \right)$$

$$s.t. \quad P(\mathbf{h}_{RB}^H \mathbf{W}^H \mathbf{W} \mathbf{h}_{RB} + \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{W} \mathbf{h}_{R1}) + \|\mathbf{W}\|_F^2 = P_R \quad (8)$$

$$\arg \max_{\mathbf{w}} \left[1 + \frac{P \mathbf{w}^H (\mathbf{h}_{R1}^T \otimes \mathbf{h}_{BR})^H (\mathbf{h}_{R1}^T \otimes \mathbf{h}_{BR}) \mathbf{w}}{\mathbf{w}^H (\mathbb{I} \otimes \mathbf{h}_{BR})^H (\mathbb{I} \otimes \mathbf{h}_{BR}) \mathbf{w} + 1} \right] \times \left[1 + \frac{P \mathbf{w}^H \mathbf{f} \mathbf{f}^H \mathbf{w}}{|h_{21}|^2 \mathbf{w}^H \mathbf{C}_1^H \mathbf{C}_1 \mathbf{w} + |h_{21}|^2 + \mathbf{w}^H \mathbf{a} \mathbf{a}^H \mathbf{w}} \right]$$

$$s.t. \quad \mathbf{w}^H [P(\mathbf{h}_{RB}^T \otimes \mathbb{I})^H (\mathbf{h}_{RB}^T \otimes \mathbb{I}) + P(\mathbf{h}_{R1}^T \otimes \mathbb{I})^H (\mathbf{h}_{R1}^T \otimes \mathbb{I}) + \mathbb{I}] \mathbf{w} = P_R \quad (9)$$

$$\mathbf{a} = (\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})^H, \quad \mathbf{C}_1 = \mathbb{I} \otimes \mathbf{h}_{2R}, \quad \mathbf{f}^H = h_{2B}(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R}) - h_{21}(\mathbf{h}_{RB}^T \otimes \mathbf{h}_{2R})$$

$$\arg \max_{\tilde{\mathbf{w}}} \frac{\tilde{\mathbf{w}}^H \left\{ \mathbf{J}^{-H} [P(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{BR})^H (\mathbf{h}_{R1}^T \otimes \mathbf{h}_{BR}) + (\mathbb{I} \otimes \mathbf{h}_{BR})^H (\mathbb{I} \otimes \mathbf{h}_{BR})] \mathbf{J}^{-1} + \frac{1}{P_R} \mathbb{I} \right\} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \left[\mathbf{J}^{-H} (\mathbb{I} \otimes \mathbf{h}_{BR})^H (\mathbb{I} \otimes \mathbf{h}_{BR}) \mathbf{J}^{-1} + \frac{1}{P_R} \mathbb{I} \right] \tilde{\mathbf{w}}}$$

$$\times \frac{\tilde{\mathbf{w}}^H \left[\mathbf{J}^{-H} (|h_{21}|^2 \mathbf{C}_1^H \mathbf{C}_1 + \mathbf{a} \mathbf{a}^H + P \mathbf{f} \mathbf{f}^H) \mathbf{J}^{-1} + \frac{|h_{21}|^2}{P_R} \mathbb{I} \right] \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \left[\mathbf{J}^{-H} (|h_{21}|^2 \mathbf{C}_1^H \mathbf{C}_1 + \mathbf{a} \mathbf{a}^H) \mathbf{J}^{-1} + \frac{|h_{21}|^2}{P_R} \mathbb{I} \right] \tilde{\mathbf{w}}} \quad (10)$$

$$s.t. \quad \tilde{\mathbf{w}}^H \tilde{\mathbf{w}} = P_R$$

We compare the sum-rate performance with respect to different relay antenna numbers. The proposed algorithms are better than the relay beamforming design targeting either SNR_1 or SINR_2 maximization and performing close to the upper bound. The performance gap between the proposals and the upper bound becomes smaller when the number of relay antenna is large. Moreover, PIA performs the closest to the tight upper bound on the sum-rate among all the three proposals and ASA has a tiny performance loss compared to PIA. Therefore, PIA is an efficient tool to address sum-rate maximization of the multi-antenna AF CDR system, although it is sub-optimal. It is also observed that the pure amplification relaying causes a significant performance loss. The sum-rate gain from the multiple-antenna relay beamforming is obvious, compared to the single antenna relay transmission. With the increase of the number of antennas at relay, we can see a clear increase in the sum-rate performance.

V. CONCLUSIONS

We focus on the relay beamforming design for sum-rate maximization of the AF CDR system. We characterize a tight upper bound on the sum-rate and propose three low-complexity but efficient algorithms to approach the achievable sum-rate maximum. Numerical results confirm that the proposals

give comparable sum-rate and perform close to the tight upper bound. PIA is identified to be the best giving near-optimal sum-rate performance. An obvious sum-rate increase from the usage of multiple relay antennas is also observed.

APPENDIX A

PROOF OF LEMMA 1

Proof: The problem (3) can be formulated in (8). It can be easily proved that the relay power constraint in (8) should be met with equality at the optimum. Therefore, it is sum-rate optimal for the relay to transmit at full power P_R . In order to rewrite the optimization cost function in a simple way, the beamforming matrix \mathbf{W} is converted into a vector form using the vectorization operation, $\mathbf{w} = \text{vec}(\mathbf{W})$. With the property $\text{vec}(\mathbf{MWN}) = (\mathbf{N}^T \otimes \mathbf{M})\text{vec}(\mathbf{W})$, we can rewrite the problem in (9) where the relay power inequality constraint is replaced by an equality constraint. We further introduce \mathbf{J} from the Cholesky decomposition

$$P(\mathbf{h}_{RB}^T \otimes \mathbb{I})^H (\mathbf{h}_{RB}^T \otimes \mathbb{I}) + P(\mathbf{h}_{R1}^T \otimes \mathbb{I})^H (\mathbf{h}_{R1}^T \otimes \mathbb{I}) + \mathbb{I} \triangleq \mathbf{J}^H \mathbf{J}.$$

We let $\tilde{\mathbf{w}} = \mathbf{J}\mathbf{w}$. When applying $\mathbf{w} = \mathbf{J}^{-1}\tilde{\mathbf{w}}$, the problem can be finally reformulated in (10). We further observe that the norm of $\tilde{\mathbf{w}}$ does not influence the maximization at all. Hence, the constraint can be ignored. This transforms the problem (10) into an unconstrained maximization problem. After some mathematical manipulations, it can be readily observed the reformulated sum-rate maximization beamforming design problem is in the form of $\arg \max_{\tilde{\mathbf{w}}} G(\tilde{\mathbf{w}}) = \arg \max_{\tilde{\mathbf{w}}} \left[\frac{\tilde{\mathbf{w}}^H \mathbf{A} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{B} \tilde{\mathbf{w}}} \times \frac{\tilde{\mathbf{w}}^H \mathbf{C} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{D} \tilde{\mathbf{w}}} \right]$ where matrices \mathbf{A} , \mathbf{B} , \mathbf{C} , and \mathbf{D} are not dependent on $\tilde{\mathbf{w}}$. We scale $\tilde{\mathbf{w}}$ in the end to fulfill the relay power constraint. This completes the proof. ■

REFERENCES

- [1] G. Farhadi and N. Beaulieu, "Capacity of amplify-and-forward multi-hop relaying systems under adaptive transmission," *IEEE Trans. Commun.*, vol. 58, no. 3, pp. 758–763, Mar. 2010.
- [2] F. Sun, T. M. Kim, A. Paulraj, E. de Carvalho, and P. Popovski, "Cell-Edge Multi-User Relaying with Overhearing," *IEEE Commun. Lett.*, vol. 17, no. 6, pp. 1160–1163, June 2013.
- [3] Y. Zhu, P.-Y. Kam, and Y. Xin, "Differential modulation for decode-and-forward multiple relay systems," *IEEE Trans. Commun.*, vol. 58, no. 1, pp. 189–199, Jan. 2010.
- [4] C. D. T. Thai, P. Popovski, E. de Carvalho, and F. Sun, "Diversity-Multiplexing Trade-off for Coordinated Direct and Relay Schemes," *IEEE Transactions on Wireless Communications*, vol. 12, no. 7, pp. 3289–3299, July 2013.

- [5] M. Uppal, Z. Liu, V. Stankovic, and Z. Xiong, "Compress-Forward Coding With BPSK Modulation for the Half-Duplex Gaussian Relay Channel," *IEEE Trans. Signal Process.*, vol. 57, no. 11, pp. 4467–4481, Nov. 2009.
- [6] P. Popovski and H. Yomo, "Bi-directional Amplification of Throughput in a Wireless Multi-Hop Network," in *Proc. IEEE VTC-Spring*, May 2006.
- [7] S. Katti, S. Gollakota, and D. Katabi, "Embracing Wireless Interference: Analog Network Coding," in *Proc. SIGCOMM*, Aug. 2007.
- [8] F. Sun and E. De Carvalho, "Degrees of Freedom of Asymmetrical Multi-Way Relay Networks," in *2011 IEEE 12th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, June 2011, pp. 531–535.
- [9] H. Ning, C. Ling, and K. K. Leung, "Wireless Network Coding with Imperfect Overhearing," *submitted to IEEE Trans. Commun.*, 2010.
- [10] H. Liu, F. Sun, C. D. T. Thai, E. De Carvalho, and P. Popovski, "Optimizing Completion Time and Energy Consumption in a Bidirectional Relay Network," in *IEEE 9th International Symposium on Wireless Communication Systems*, Aug 2012, pp. 999–1003.
- [11] H. Liu, P. Popovski, E. de Carvalho, Y. Zhao, and F. Sun, "Four-Way Relaying in Wireless Cellular Systems," *IEEE Wireless Commun. Lett.*, vol. 2, no. 4, pp. 403–406, August 2013.
- [12] H. Liu, F. Sun, E. de Carvalho, P. Popovski, H. Thomsen, and Y. Zhao, "MIMO Four-Way Relaying," in *2013 IEEE 14th Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, June 2013, pp. 46–50.
- [13] F. Sun, E. De Carvalho, C. D. T. Thai, and P. Popovski, "Beamforming design for coordinated direct and relay systems," in *2012 46th Annual Conference on Information Sciences and Systems (CISS)*, March 2012, pp. 1–6.
- [14] F. Sun, E. De Carvalho, P. Popovski, and C. D. T. Thai, "Coordinated Direct and Relay Transmission With Linear Non-Regenerative Relay Beamforming," *IEEE Signal Processing Letters*, vol. 19, no. 10, pp. 680–683, Oct. 2012.
- [15] B. Bandemer, Q. Li, X. Lin, and A. Paulraj, "Overhearing-Based Interference Cancellation for Relay Networks," in *Proc. IEEE VTC-Fall*, Sep. 2009.
- [16] H. Yomo and E. Carvalho, "Spectral efficiency enhancement with interference cancellation for wireless relay network," in *Proc. IEEE PIMRC*, Sep. 2008.
- [17] W. Chen, K. Letaief, and Z. Cao, "Network interference cancellation," *IEEE Trans. Commun.*, vol. 8, no. 12, pp. 5982–5999, Dec. 2009.
- [18] T. Brown, E. D. Carvalho, and P. Kyritsi, *Practical guide to the MIMO Radio Channel: with MATLAB Examples*. Wiley, 2012.
- [19] F. Sun, P. Popovski, C. D. T. Thai, and E. de Carvalho, "Sum-rate maximization of coordinated direct and relay systems," in *2012 18th European Wireless Conference European Wireless (EW)*, April 2012, pp. 1–7.
- [20] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 5, pp. 699–712, June 2009.
- [21] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, March 2004.
- [22] S. Boyd, "Branch and Bound Methods," available online, 2007.
- [23] W. Cheng, M. Ghogho, Q. Huang, D. Ma, and J. Wei, "Maximizing the Sum-Rate of Amplify-and-Forward Two-Way Relaying Networks," *IEEE Signal Process. Lett.*, vol. 18, no. 11, pp. 635–638, Nov. 2011.
- [24] S. K. Joshi, P. C. Weeraddana, M. Codreanu, and M. Latva-aho, "Weighted Sum-Rate Maximization for MISO Downlink Cellular Networks via Branch and Bound," *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 2090–2095, Apr. 2012.

- [25] I. Karasalo, “Estimating the covariance matrix by signal subspace averaging,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 34, no. 1, pp. 8–12, Feb. 1986.
- [26] L. Chen, K.-K. Wong, H. Chen, J. Liu, and G. Zheng, “Optimizing Transmitter-Receiver Collaborative-Relay Beamforming with Perfect CSI,” *IEEE Commun. Lett.*, vol. 15, no. 3, pp. 314–316, Mar. 2011.
- [27] J. C. Lagarias, J. A. Reeds, M. H. Wright, and P. E. Wright, “Convergence Properties of the Nelder–Mead Simplex Method in Low Dimensions,” *SIAM J. Optim.*, vol. 9, no. 1, pp. 112–147, Feb. 1998.
- [28] N. Lee, H. J. Yang, and J. Chun, “Achievable Sum-Rate Maximizing AF Relay Beamforming Scheme in Two-Way Relay Channels,” in *Proc. IEEE ICC*, May 2008.
- [29] G. H. Golub and C. F. V. Loan, *Matrix Computations*. The Johns Hopkins University Press, Third Edition, 1996.